Fixed vs. Random Factors

Fixed:

- All levels of interest are included in the experiment.
- Interest is in *means*.
- Treatment effects sum to 0.
- Inference is over levels in the experiment.

Random:

- Levels are randomly selected from larger population.
- Interest is in variances.
- Treatment effects are normal $(0, \sigma^2)$.
- Inference is over levels in population.

Three Model Types

<u>Fixed</u> – all factors and interactions are fixed effects (except residual error)

<u>Random</u> – all factors and interactions are random (no fixed factors in model)

Mixed – contains at least one fixed factor and two or more random factors (including residual) ∴ contains mixed interactions between fixed and random factors

Expected Mean Squares Algorithm Setting up the Table

- 1. Write the linear model for the experiment.
- Create a table with c columns where c = 2 + the # subscripts and r rows where r = number of terms in model.
- 3. Label the column on left **Source**, the one to the far right, **EMS**. The remaining columns in between are labeled with subscripts used in the model (eg. ijk).
- 4. In the column heading above each subscript use an F or R to indicate whether the factor associated with it is fixed or random.
- 5. Above these, write the number of levels associated with the subscript.
- Enter sources of variation from the model in left-hand column, excluding the mean and including subscripts for each term.
- 7. In the subscript columns:
 - a. Enter a 1 in each cell where the column intersects a row where the subscript occurs in parentheses.
 - b. Enter a 1 in each cell where a column labeled as random intersects a row where the subscript associated with the column occurs in the term.
 - c. Enter a 0 in each cell where a column labeled as **fixed** intersects a row where the subscript associated with the column occurs in the term.
 - d. Enter the number of treatment levels associated with the column in all remaining cells.

Expected Mean Squares Algorithm Working the Algorithm

- 1. Beginning with the last row in the table, construct the expected mean square for each factor listed in the table by:
 - a. Only consider including variance components for model terms with all of the subscripts for the factor in question.
 - b. To compute the coefficient for a term, multiply the coefficients in the cells to the right of the term **excluding** the cells associated with subscripts included in the term.
 - c. If the product is zero, then that term is not included in the variance components for the factor under consideration.
 - d. If the product is greater than zero, then that term is included as a variance component in the expected mean square for the factor under consideration.
 - e. Write the EMS as a linear combination of variance components in right-hand column using σ to denote random effects and ϕ to denote fixed effects.
- 2. Continue the steps above for each model term in the left column always working from the bottom upwards.

Model:
$$Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \epsilon_{(ij)k}$$

Factors:

Factor A – Fixed Factor B – Fixed Reps - Random

Expected Mean Squares Algorithm

Create table and enter sources of variation from the model.

Source	i	j	k	EMS
A _i				
B _j				
AB _{ij}				
$\epsilon_{(ij)k}$				

Enter the number of levels and indicate if fixed or random effect above subscripts.

Source	a F i	b F j	r R k	EMS
A _i				
B _j				
AB _{ij}				
$\epsilon_{(ij)k}$				

Expected Mean Squares Algorithm

Enter a 1 in each cell where subscript is contained in parentheses in model term.

Source	a F i	b F i	r R k	EMS
A _i	-	,		
B _j				
AB _{ij}				
$\epsilon_{(ij)k}$	1	1		

Enter a 0 in cells where the subscripts correspond to a fixed term.

Source	a F i	b F j	r R k	EMS
A _i	0			
B _j		0		
AB _{ij}	0	0		
ε _{(ij)k}	1	1		

Expected Mean Squares Algorithm

Enter a 1 in cells where the subscripts correspond to a random term.

	а	b	r	
	F	F	R	
Source	i	j	k	EMS
A _i	0			
B _j		0		
AB _{ij}	0	0		
ε _{(ij)k}	1	1	1	

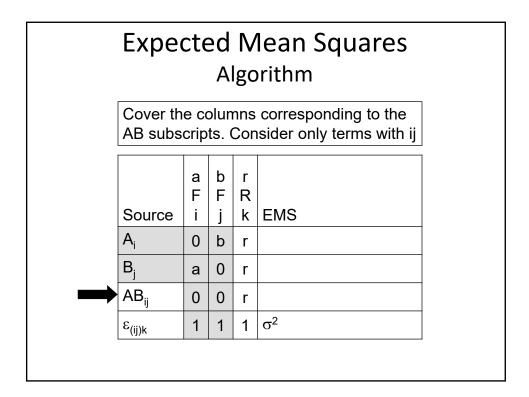
Fill remaining cells with number of levels corresponding to subscript.

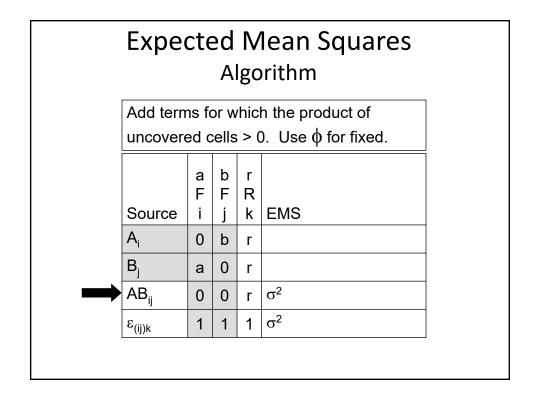
	а	b	r	
	F	F	R	
Source	i	j	k	EMS
A _i	0	b	r	
B _j	а	0	r	
AB _{ij}	0	0	r	
ε _{(ij)k}	1	1	1	

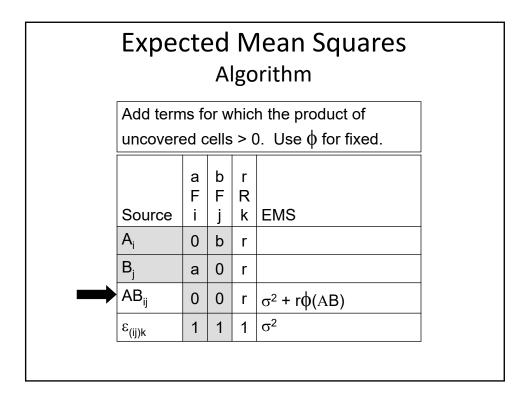
Expected Mean Squares Algorithm

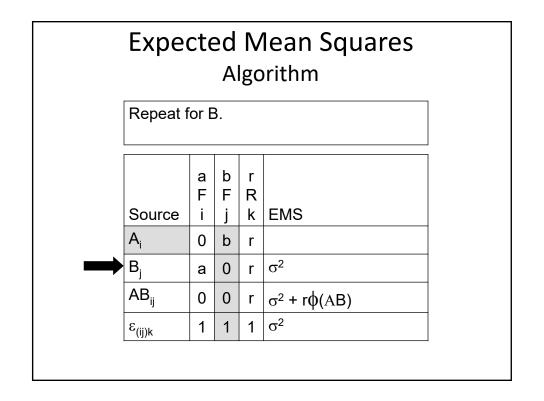
Enter σ^2 in the bottom cell on the right. This is the error variance.

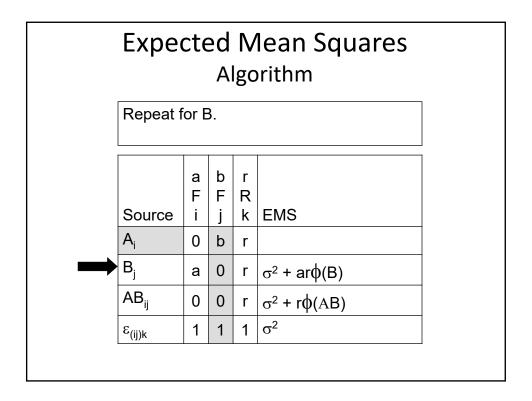
		a F	b F	r R	
	Source	i	j	k	EMS
	A _i	0	b	r	
	B _j	а	0	r	
	AB _{ij}	0	0	r	
>	ε _{(ij)k}	1	1	1	σ^2

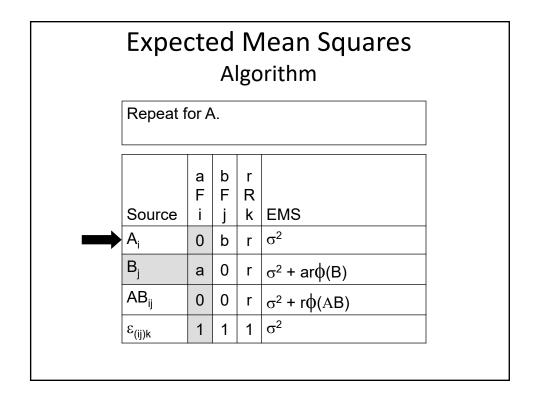


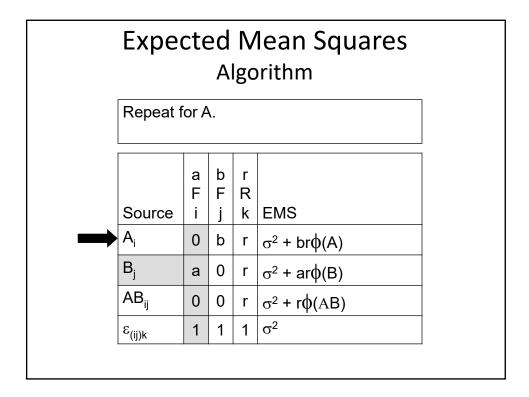


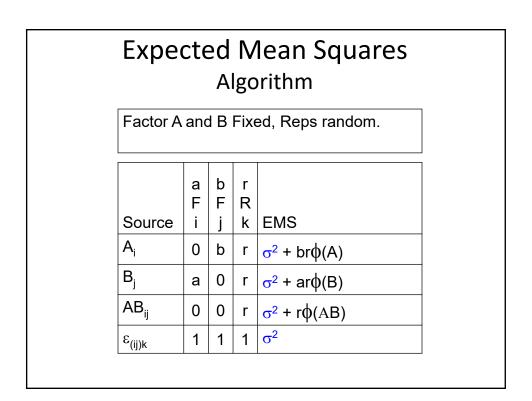












Factor A fixed. Factor B and Reps random.

	а	b	r	
	F	R	R	
Source	i	j	k	EMS
A _i	0	b	r	
B _j	а	1	r	
AB _{ij}	0	1	r	
$\epsilon_{(ij)k}$	1	1	1	

Expected Mean Squares Algorithm

Factor A fixed. Factor B and Reps random.

	а	b	r	
	F	R	R	
Source	i	j	k	EMS
A _i	0	b	r	$\sigma^2 + r\sigma^2_{AB} + br\phi(A)$
B _j	а	1	r	σ^2 + ar σ^2_B
AB _{ij}	0	1	r	$\sigma^2 + r\sigma^2_{AB}$
ε _{(ij)k}	1	1	1	σ^2

Factor A fixed. Factor B and Reps random.

	a F	b R	r R	
Source	i	j	k	EMS
A _i	0	b	r	$\sigma^2 + r\sigma^2_{AB} + br\phi(A)$
B _j	а	1	r	σ^2 + ar σ^2_B
AB _{ij}	0	1	r	$\sigma^2 + r\sigma^2_{AB}$
ε _{(ij)k}	1	1	1	σ^2

Expected Mean Squares Algorithm

Factors A, B and Reps random.

	a R	b R	r R	
Source	i	j	k	EMS
A _i	1	b	r	
B _j	а	1	r	
AB _{ij}	1	1	r	
$\epsilon_{(ij)k}$	1	1	1	

Factors A, B and Reps random.

	а	b	r	
	R	R	K	
Source	i	j	k	EMS
A _i	1	b	r	$\sigma^2 + r\sigma^2_{AB} + br\sigma^2_{A}$
B _j	а	1	r	$\sigma^2 + r\sigma^2_{AB} + ar\sigma^2_{B}$
AB _{ij}	1	1	r	$\sigma^2 + r\sigma^2_{AB}$
$\epsilon_{(ij)k}$	1	1	1	σ^2

Expected Mean Squares Algorithm

Factors A, B and Reps random.

	a R	b R	r R	
Source	i	j	k	EMS
A _i	1	b	r	$\sigma^2 + r\sigma^2_{AB} + br\sigma^2_{A}$
B _j	а	1	r	$\sigma^2 + r\sigma^2_{AB} + ar\sigma^2_{B}$
AB _{ij}	1	1	r	$\sigma^2 + r\sigma^2_{AB}$
ε _{(ij)k}	1	1	1	σ^2

Expected Mean Squares 3-Factor ANOVA

Model:

$$Y_{ijkl} = \mu + A_i + B_j + AB_{ij} + C_k + AC_{ik} + BC_{jk} + ABC_{ijk} + R_{(ijk)l}$$

Factors:

- A 2
- B 4
- C 3
- R 2

Expected Mean Squares

Direct Tests

A,B, and C Fixed Factors

Source	df	EMS
A _i 1		$\sigma^2 + 24\phi(A)$
B_{i}	3	$\sigma^2 + 12\phi(B)$
AB_{ij}	3	$\sigma^2 + 6\phi(AB)$
C_k	2	$\sigma^2 + 16\phi(C)$
AC _{ik}	2	$\sigma^2 + 8\phi(AC)$
BC_{jk}	6	$\sigma^2 + 4\phi(BC)$
$\overline{ABC_{ijk}}$	6	$\sigma^2 + 2\phi(ABC)$
$\varepsilon_{(ijk)l}$	24	σ^2

Expected Mean Squares

Direct Tests

A Fixed, B and C Random Factors

A likeu, Ballu C Kalluolii l'actors				
Source	df	EMS		
A_{i}	1	$\sigma^2 + 2\sigma^2_{ABC} + 8\sigma^2_{AC} + 6\sigma^2_{AB} + 24\phi(A)$		
B _i	3	$\sigma^2 + 4\sigma_{BC}^2 + 12\sigma_{B}^2$		
AB_{ij}	3	$\sigma^2 + 2\sigma^2_{ABC} + 6\sigma^2_{AB}$		
C_k	2	$\sigma^2 + 4\sigma^2_{BC} + 16\sigma^2_{C}$		
AC_{ik}	2	$\sigma^2 + 2\sigma^2_{ABC} + 8\sigma^2_{AC}$		
BC_{ik}	6	$\sigma^2 + 4\sigma^2_{BC}$		
ABC_{ijk}	6	$\sigma^2 + 2\sigma^2_{ABC}$		
$\varepsilon_{(ijk)l}$	24	σ^2		

Expected Mean Squares

Direct Tests

A Fixed, B and C Random Factors

7 Thea, Bana e Hanaem ractors				
Source	df	EMS		
A_{i}	1	$\sigma^2 + 2\sigma^2_{ABC} + 8\sigma^2_{AC} + 6\sigma^2_{AB} + 24\phi(A)$		
\mathbf{B}_{i}	3	$\sigma^2 + 4\sigma^2_{BC} + 12\sigma^2_{B}$		
AB_{ij}	3	$\sigma^2 + 2\sigma^2_{ABC} + 6\sigma^2_{AB}$		
C_k	2	$\sigma^2 + 4\sigma^2_{BC} + 16\sigma^2_{C}$		
AC _{ik}	2	$\sigma^2 + 2\sigma^2_{ABC} + 8\sigma^2_{AC}$		
BC_{jk}	6	$\sigma^2 + 4\sigma^2_{BC}$		
ABC_{ijk}	6	$\sigma^2 + 2\sigma^2_{ABC}$		
$\varepsilon_{(ijk)l}$	24	σ^2		

Expected Mean Squares

Approximate Tests

A Fixed, B and C Random Factors

h(A)
4(1)
φ(A)

Approximate F test: MS(A)/[MS(AB)+MS(AC)-MS(ABC)]

Approximate F Tests Satterthwaite's Approximation

$$df = \frac{M^{2}}{\frac{(a_{1}MS_{1})^{2}}{df_{1}} + \frac{(a_{2}MS_{2})^{2}}{df_{2}} + ... + \frac{(a_{k}MS_{k})^{2}}{df_{k}}}$$

where:
$$M = \sum_{i=1}^{k} a_i M S_i$$

Example:

$$df = \frac{[MS(AB) + MS(AC) - MS(ABC)]^{2}}{MS(AB)^{2} / 3 + MS(AC)^{2} / 2 + MS(ABC)^{2} / 6}$$

Expected Mean Squares

Conservative Tests

A Fixed, B and C Random Factors

7 Tixed, Band e Random ractors				
Source	df	EMS		
A_{i}	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 3\sigma^2_{AB} + 12\phi(A)$		
B _i	3	$\sigma^2 + 2\sigma^2_{BC} + 6\sigma^2_{B}$		
AB_{ij}	3	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{AB}$		
C_k	2	$\sigma^2 + 2\sigma^2_{BC} + 8\sigma^2_{C}$		
AC_{ik}	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$		
BC_{jk}	6	$\sigma^2 + 2\sigma^2_{BC}$		
ABC_{ijk}	6	$\sigma^2 + \sigma^2_{ABC}$		
$\epsilon_{(ijk)l}$	0	σ^2		